Government of **Western Australia**
School Curriculum and Standards Authority

MATHEMATICS APPLICATIONS

Calculator-assumed

Sample WACE Examination 2016

Marking Key

Marking keys are an explicit statement about what the examiner expects of candidates when they respond to a question. They are essential to fair assessment because their proper construction underpins reliability and validity.

 2014/10407

Section Two: Calculator-assumed **35% (96 Marks)**

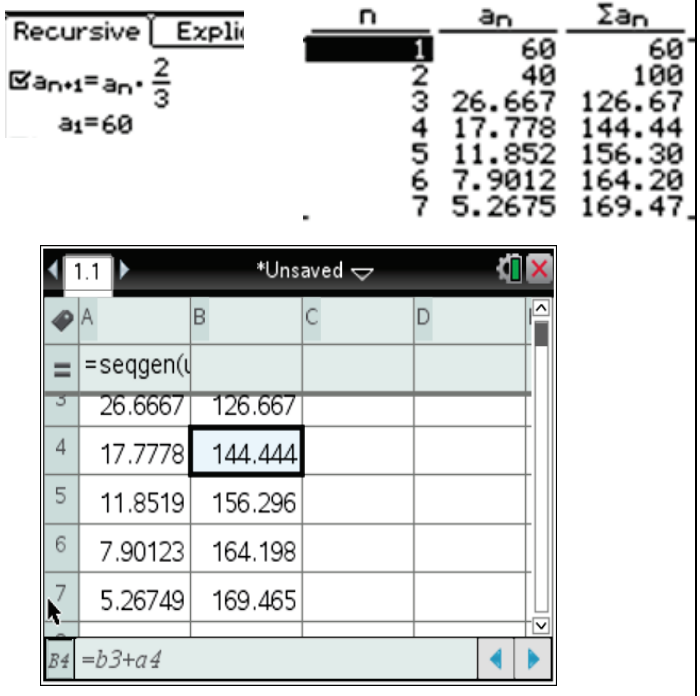
Question 9 **(7 marks)**

A ball is dropped from a height of 60 cm onto a horizontal surface. The height reached by the ball after each bounce is two-thirds of the height of the previous bounce.

(a) Write a recursive rule to show the distance travelled by each successive downward motion of the ball. (2 marks)

Solution
$T_{n+1} = \frac{2}{3}T_n \ , T_1 = 60$
Specific behaviours
✓ correctly writes a recursive rule ✓ correctly states first term

(b) What is the maximum height reached by the ball after the third bounce, correct to **two** decimal places? (2 marks)

Solution
T_4 = 17.78 cm 
Specific behaviours
✓ states the correct term ✓ states answer correct to two decimal places

The coefficient of restitution e is a measure of the bounciness of an object such as a ball on impact with the floor. This can be measured by the ratio

$e = \dfrac{H_{n+1}}{H_n}$ where H_n= the height of a given bounce and $0 \le e < 1$

(c) (i) Calculate the coefficient of restitution for the ball in part (a). (1 mark)

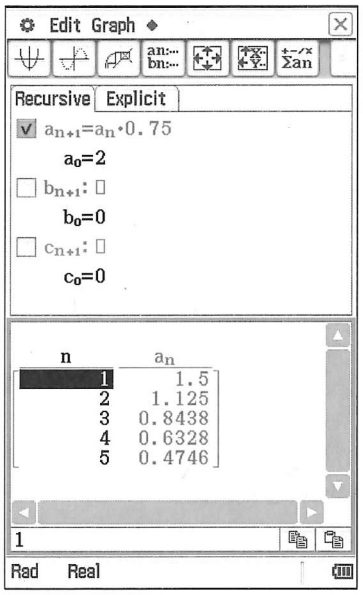
(ii) Given $e = 0.75$ calculate to the nearest centimetre, the maximum height of the third bounce of a ball dropped from 2 metres. (2 marks)

Solution

(i) $e = \dfrac{2}{3}$

$0.75 = \dfrac{H_{n+1}}{H_n}$ where $H_0 = 2.00\ m$

(ii)
$\Rightarrow H_1 = 0.75 \times 2.00 = 1.50\ m$
$\Rightarrow H_2 = 0.75 \times 1.50 = 1.125\ m$
$\Rightarrow H_3 = 0.75 \times 1.125 = 0.84\ m \ \ = 84\ cm$



Specific behaviours

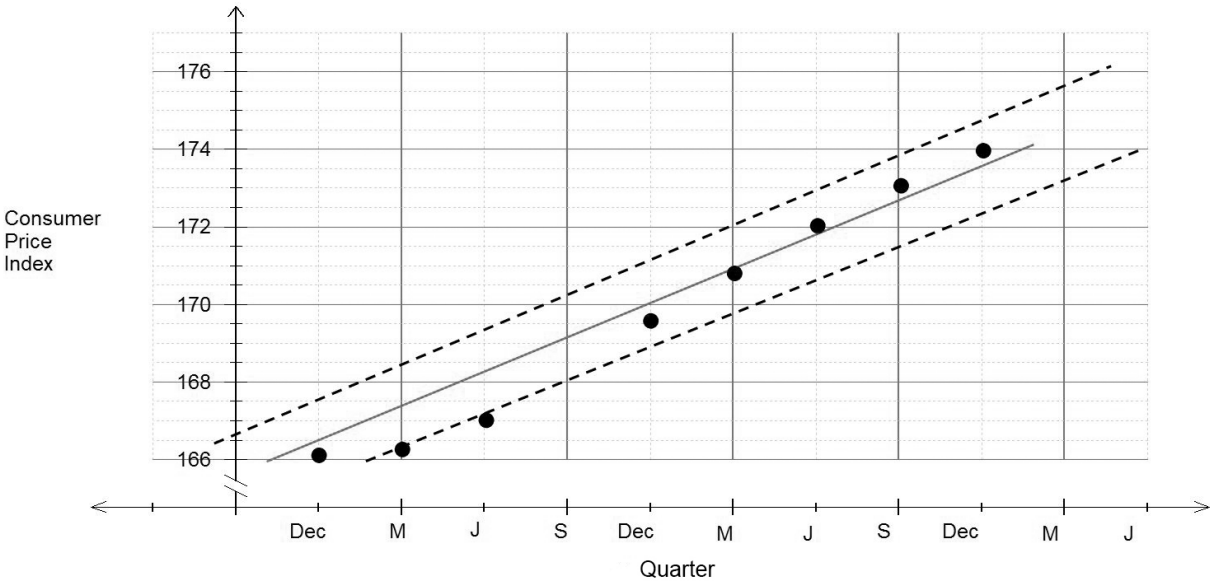
✓ states the correct value for e
✓ sets up the correct recurrence relation
✓ calculates the value for H_3 correct to the nearest centimetre

Question 10 **(7 marks)**

The Consumer Price Index (CPI) is measured every three months and is used to help determine inflation rates. The following information has been provided by the Australian Bureau of Statistics.

Quarter	**Dec 2008**	**Mar 2009**	**June 2009**	**Sept 2009**	**Dec 2009**	**Mar 2010**	**June 2010**	**Sept 2010**	**Dec 2010**	**Mar 2010**	**June 2011**
Month (*m*)	1	4	7	10	13	16	19	22	25	28	31
Consumer Price Index (*i*)	166.0	166.2	167.0	n.a.	169.5	171.0	172.1	173.3	174.0	n.a.	n.a.

(a) Plot the above data on the axes below. (2 marks)



Solution
As shown on grid.
Specific behaviours
✓ plots at least five points correctly ✓ plots at least seven points correctly

(b) Determine the equation of the least squares line that models the relationship between the time in months and the CPI for the data provided. (1 mark)

Solution
$CPI = 0.3658m + 164.99$ or $y = 0.3658x + 164.99$
Specific behaviours
✓ determines least squares line equation

(c) Use your line from part (b) to predict the CPI for September 2009. (1 mark)

Solution
168.6
Specific behaviours
✓ correct prediction from regression line

(d) Which would be the more accurate prediction: The CPI for September 2009 or the CPI for September 2011? Justify your choice of answer. (2 marks)

Solution
CPI for September 2009 there is no guarantee that the current trend will continue OR the prediction for September 2009 is from within the time period provided (interpolation) whereas a prediction for September 2011, involves extrapolation which is less reliable.
Specific behaviours
✓ correctly states September 2009 ✓ justifies choice

(e) The data show an increasing trend in the value of the CPI over time. What does the equation, as determined in part (b), indicate is the rate of increase in the CPI? (1 mark)

Solution
CPI rises at a rate of 0.3658 every month
Specific behaviours
✓ identifies the correct rate per month

Question 11 **(8 marks)**

(a) The results of a survey of students' first preferences for work placements are given in the table below.

	Monday	Tuesday	Wednesday
Kindergarten	42	48	43
Hospital	23	24	26
Nursing home	14	12	13

(i) If Kindergarten placements are allocated to Mondays, Hospital placements are allocated to Tuesdays and Nursing home placements are allocated to Wednesdays, how many students will receive their first preferences? (1 mark)

Solution
$42 + 24 + 13 = 79$
Specific behaviours
✓ calculates the correct total

(ii) Identify the maximum number of students who can receive their first preference by reallocating each work place to a different day of the week. (1 mark)

Solution
Row 1 =48, Row 2 =26 and row 3=14 Total =88 first preferences.
Specific behaviours
✓ Calculates the correct total

(b) An extra work place has become available. Thursdays were included in the roster and a new survey for first preferences was conducted with the results shown in the table below.

	Monday	Tuesday	Wednesday	Thursday
Kindergarten	23	21	19	13
Hospital	21	19	18	10
Nursing home	13	13	9	12
Day care centre	15	15	19	13

The Hungarian algorithm is to be used to identify the maximum number of students who could get their first preference by allocating each work place to a particular day of the week.

(i) The first step of the algorithm has been partially completed. Complete the table below to show the result of this process. (1 mark)

Solution

	Monday	Tuesday	Wednesday	Thursday
Kindergarten	0	2	4	10
Hospital	2	4	5	13
Nursing home	10	10	14	11
Day care centre	8	8	4	10

Specific behaviours

✓ correctly calculates elements of the row

At a later stage in using the Hungarian algorithm, the table was presented as follows:

	Monday	Tuesday	Wednesday	Thursday
Kindergarten	~~0~~	2	4	9
Hospital	~~0~~	2	3	10
Nursing home	~~0~~	~~0~~	~~4~~	~~0~~
Day care centre	~~4~~	~~4~~	~~0~~	~~5~~

(ii) Complete the next step of the algorithm in the table below (3 marks)

Solution

	Monday	Tuesday	Wednesday	Thursday
Kindergarten	0	0	2	7
Hospital	0	0	1	8
Nursing home	2	0	4	0
Day care centre	6	4	0	5

Specific behaviours

✓ completes the subtraction of the smallest number from the uncovered cells
✓completes the addition of the smallest number at the intersection cells
✓completes the table correctly

(iii) Allocate the work places to the correct days to maximise the first preferences by shading the correct cells in the table below. (2 marks)

Solution

	Monday	Tuesday	Wednesday	Thursday
Kindergarten	23	21	19	13
Hospital	21	19	18	10
Nursing home	13	13	9	12
Day care centre	15	15	19	13

Specific behaviours

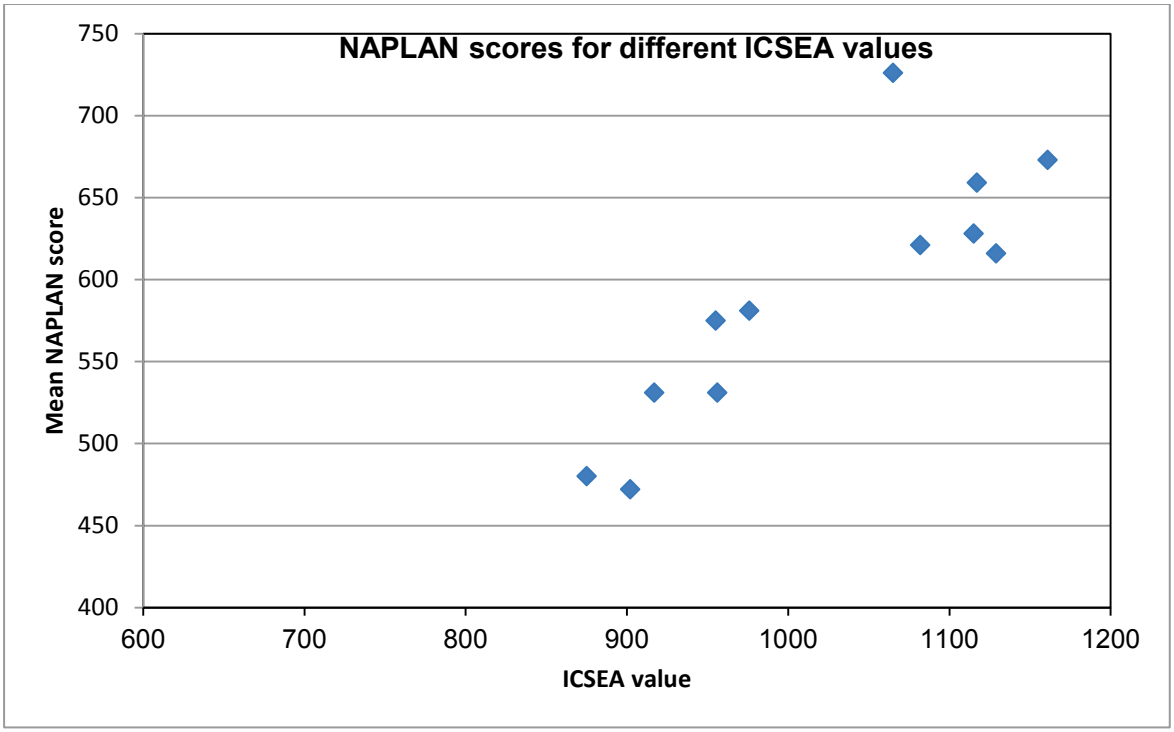
✓ allocates at least 3 cells correctly
✓ allocates all four cells correctly

Question 12 **(5 marks)**

The following figures were obtained from the My School website. The second column shows the ICSEA value for the school (an indication of the socioeconomic circumstances of the school population) and the third column shows the mean score for the school on the 2008 NAPLAN numeracy test for Year 9 students in that school.

School	ICSEA value (x)	Mean NAPLAN score Year 9 Numeracy (y)
A	1117	659
B	1082	621
C	875	480
D	955	575
E	1129	616
F	917	531
G	1065	726
H	976	581
I	902	472
J	956	531
K	1115	628
L	1161	673

From the scatterplot of the data:



(a) (i) Identify the response variable. (1 mark)

Solution
The mean NAPLAN score
Specific behaviours
✓ identifies the response variable

(ii) Describe the apparent relationship between the two variables. (1 mark)

Solution
The higher the ICSEA value for the school the greater the mean NAPLAN score for Year 9 numeracy
Specific behaviours
✓ determines the relationship between the two variables

(b) Calculate the correlation coefficient r_{xy}. (1 mark)

Solution
$r_{xy} = 0.86$ or (0.8576)
Specific behaviours
✓ determines the value of the correlation coefficient

(c) Calculate the coefficient of determination. (1 mark)

Solution
$r^2 = 0.7354$
Specific behaviours
✓ determines the value of the coefficient of determination

(d) What percentage of the variation in mean NAPLAN scores between these schools can be explained by the variation in ICSEA values? (1 mark)

Solution
73.5%
Specific behaviours
✓ determines the percentage of variation

Question 13 **(11 marks)**

To save money to travel overseas, Luke started an investment account. He used $1000 to open the account and then deposited an extra $200 at the end of each month for two years.
The table below shows the following:

- The amount in the account at the beginning of each month (A_n)
- The interest added to the account each month (I_n)
- The deposit made near the end of each month (D_n)
- The amount in the account at the end of each month (A_{n+1}).

Month (n)	Amount at beginning of month (A_n)	Calculation of interest for month (I_n)	Deposit for month (D_n)	Amount at end of month (A_{n+1})
1	$1000.00	$1000.00 x 1.012	$200.00	$1212.00
2	$1212.00	$1212.00 x 1.012	$200.00	$1426.54
3	$1426.54	$1426.54 x 1.012	$200.00	$1643.66
4	$1643.66	$1643.66 x 1.012	$200.00	$1863.39
5	$1863.39	$1863.39 x 1.102	$200.00	$2085.75
6	$2085.75	$2085.75 x 1.012	$200.00	$2310.78

Note: The values in this table have been rounded to two decimal places.

(a) What is the monthly interest rate? (1 mark)

Solution
1.2%
Specific behaviours
✓ determines the monthly interest rate

(b) Write a recursive rule to calculate the amount in the account at the end of each month. (2 marks)

Solution
$A_{n+1} = A_n\,(1.012) + 200,\ A_1 = 1000$
Specific behaviours
✓ defines recursive rule correctly ✓ defines initial term

(c) How much is in the account after two years? (2 marks)

Solution
$6856
Specific behaviours
✓ assigns 25 to n ✓ calculates balance after two years correctly

(d) How much interest did Luke receive over the two-year period? (2 marks)

Solution
$\$6856 - 1000 - 200 \times 24 = \$1056$
Specific behaviours
✓ subtracts initial amount and monthly deposit correctly ✓ calculates total interest correctly

(e) For many investment accounts, interest is calculated daily but paid into the account monthly. Would this process have given Luke a higher or lower amount of interest over the two years? Explain your choice of answer. (2 marks)

Solution
Higher. The interest calculation each day is being based on a higher amount so the amount of interest grows more quickly.
Specific behaviours
✓ identifies a higher amount of interest for two year period ✓ explains that interest calculation is based on higher amount in the account

(f) If Luke had been offered double the rate of interest but still opened his investment account with \$1000 and deposited \$200 each month would the interest earned on his investment over the two-year period have been:

A twice as much
B less than twice as much
C more than twice as much.

State your choice and give a reason for your answer. (2 marks)

Solution
More than twice as much. For the first month the interest is doubled but after that the basis for the calculation of interest is greater so more than twice as much interest is earned each time. Alternatively students may provide data for 1.2% \$1056 is earned and for 2.4% \$2357.24 is earned and this is more than twice \$1056 (\$2112).
Specific behaviours
✓ identifies option C, more than twice as much ✓ describes compounding process or quote interest values from data

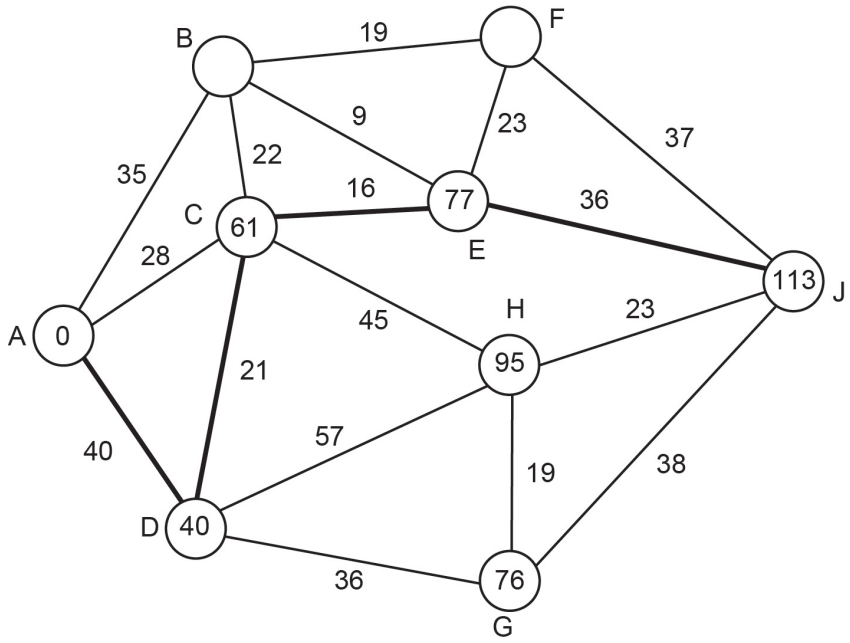
Question 14 **(8 marks)**

A local park has nine main attractions. The paths between these attractions are shown on the network below, along with the distances, in metres, between each attraction.

Jason is at the children's playground A and Sandra is at the barbecue area D. Sandra calls Jason on his mobile phone and says that she wants to go with him to the waterfall J. She will wait for him at the barbecue area D.

(a) What is the shortest way for Jason to travel from the children's playground A to the waterfall J, joining Sandra at the barbecue area D on the way? State both the path and the distance. (3 marks)

Solution



Path ADCEJ
Distance 113 metres

Specific behaviours

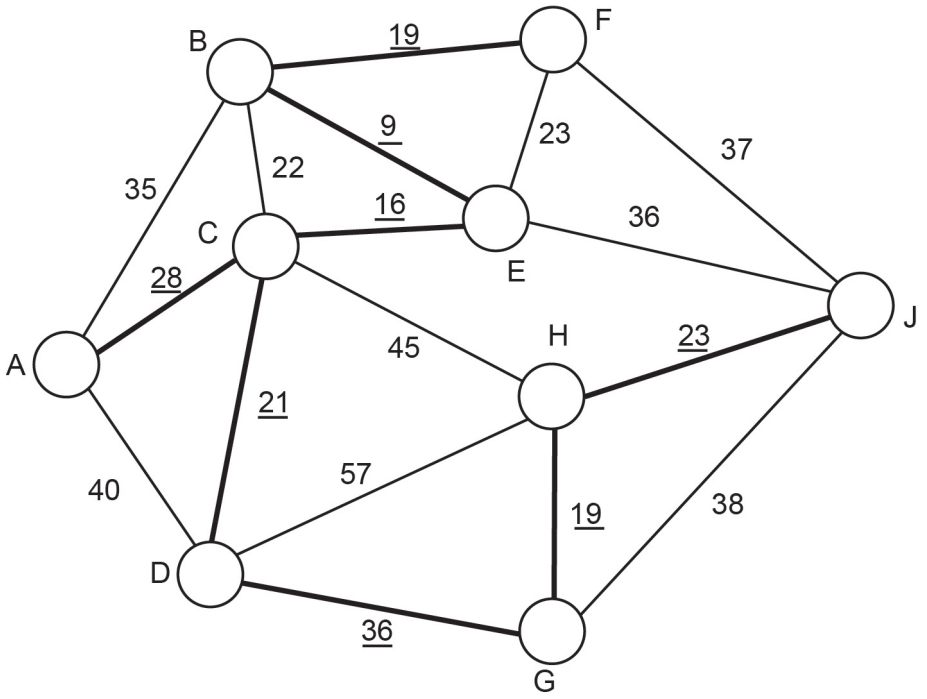
- ✓ identifies D as the starting point with 40 metres already walked
- ✓ states the path correctly
- ✓ states the distance of the path correctly
  (except the trivial case of 80 m for path ACEJ)

(b) The local council has decided to resurface some of the connections between the nine attractions. They want every attraction to be connected so that wheelchairs can move between them. The new connections will be expensive to resurface and the cost is to be kept to a minimum.

(i) On the diagram below show the connection(s) that the council should resurface. (2 marks)

(ii) What is the total length of the connections to be resurfaced? (1 mark)

Solution



(i) Path = 19 + 9 + 16 + 28 + 21 + 36 + 19 + 23

(ii) 171 m

Specific behaviours

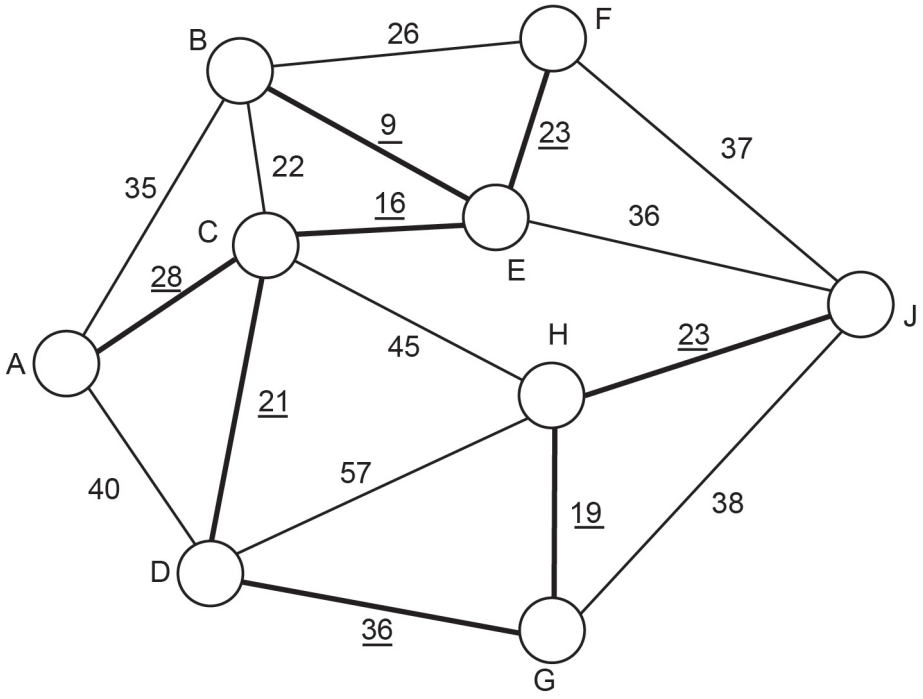
✓ ✓ states/draws correct paths identified through a spanning tree
✓ states correct distance

or

✓ draws the correct connections through the network
✓ draws a spanning tree, not a cycle
✓ states the correct distance

(c) Before the resurfacing began, the connection between attractions B and F had to be altered to 26 m in length. How did this alteration change the connections to be resurfaced and the total length? (2 marks)

Solution



New path as shown in bold: $9 + 23 + 16 + 28 + 21 + 36 + 19 + 23 = 175$ m

Specific behaviours

✓ states new path
✓ calculates length correctly

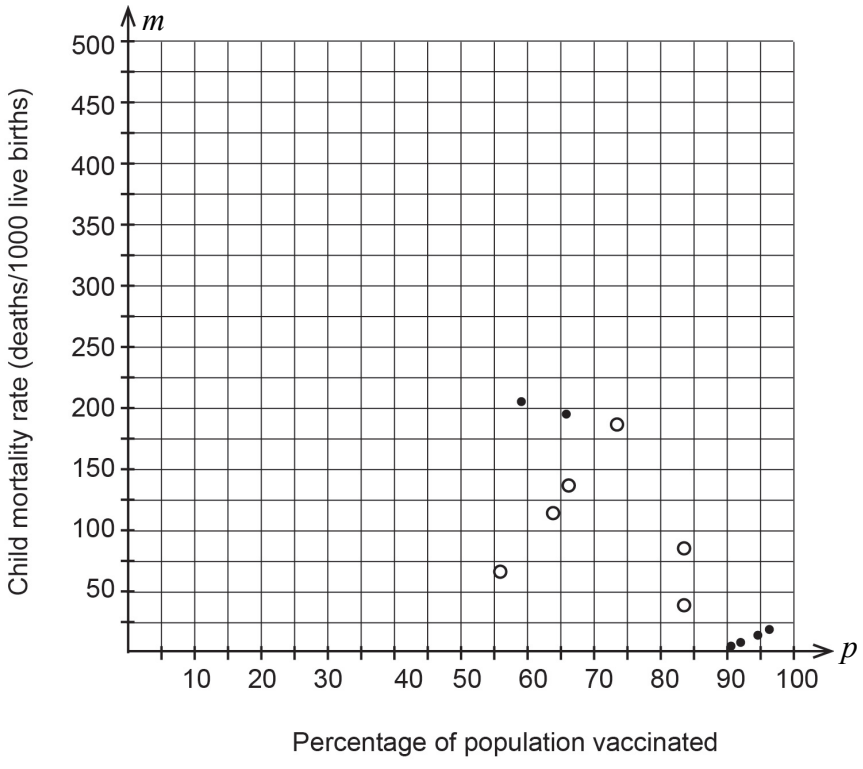
Question 15 **(6 marks)**

The World Health Organisation (WHO) monitors the percentage of a country's target population that has been vaccinated against a number of diseases as well as the child (under 5 years) mortality rate for that country. The table below shows the percentage of the target population vaccinated against hepatitis B and the child mortality rate (deaths per 1000 live births) for 12 countries (A–L).

Country	Percentage of target population vaccinated against Hepatitis B (p)	Child (under 5 years) mortality rate/1000 live births (m)
A	92	5
B	66	199
C	59	209
D	94	15
E	96	21
F	90	4
G	83	84
H	56	68
I	66	138
J	73	191
K	64	112
L	83	39

(a) The information for the first six countries (A to F) has been plotted on the scatterplot below. Complete the scatterplot. (2 marks)

Solution



Specific behaviours

✓ plots at least four points correctly
✓ plots all points correctly

The equation for the least-squares line that models the relationship between the percentage of the target population vaccinated (p) and the child mortality rate (deaths per 1000 live births) (m) is $m = -4.0808p + 403.959$ and the correlation coefficient is $r_{pm} = -0.7565$.

(b) Predict the child mortality rate where only a quarter of the target population has been vaccinated. (2 marks)

Solution
$p = 25,\ m = -4.0808(25) + 403.959 = 301.9$ (accept 302)
Specific behaviours
✓ identifies p value $= 25$ ✓ calculates the predicted mortality rate for a quarter of the population

(c) Comment on the reliability of your prediction from part (b). Give one reason to justify your answer. (2 marks)

Solution
It is not reliable since (i) it is well outside the range of the data provided and on which the trend line is based **or** reliability is reduced when extrapolating OR (ii) the correlation value is not very high, not near –1 or 1
Specific behaviours
✓ determines prediction is not reliable ✓ states one of these reasons to justify conclusion

Question 16 **(7 marks)**

Karen borrowed \$20 000 to purchase a new car. Interest on the loan was set at 8.5% per annum. If the interest of the loan (*\$A)* is compounded n times per year, the amount owed (*\$V)* after one year is given by $V = A\left(1+\frac{r}{100n}\right)^n$.

(a) State the value of A. (1 mark)

Solution
$A = \$20\,000$
Specific behaviours
✓ determines correct value for A

(b) State the value of r. (1 mark)

Solution
$r = 8.5\%$
Specific behaviours
✓ determines correct value for interest rate

(c) Calculate V when $n = 4$. (2 marks)

Solution
$V = 20\,000\left(1+\frac{8.5}{400}\right)^4 = \$21\,754.96$
Specific behaviours
✓ substitutes $n = 4$ correctly into the formula for V ✓ calculates V correctly

(d) Calculate the effective annual rate of interest when interest is compounded monthly. Give your answer as a percentage correct to **two** decimal places. (3 marks)

Solution
$i_e = (1 + 8.5 \div 100 \div 12)^{12} - 1 = 0.08839 = 8.84\%$
Specific behaviours
✓ substitutes $n = 12$ into effective interest formula ✓ calculates correct effective annual interest rate ✓ states interest rate correct to two decimal places

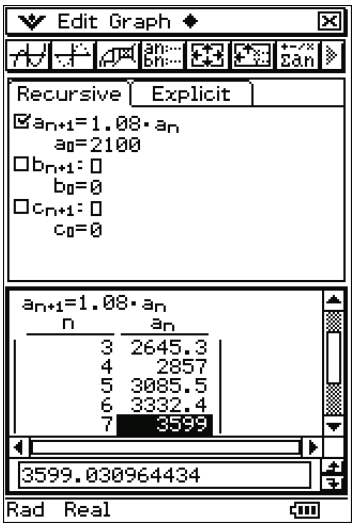
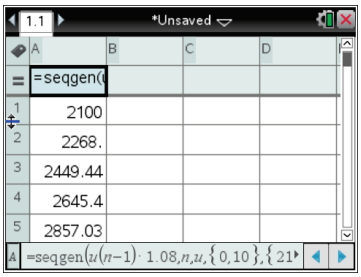
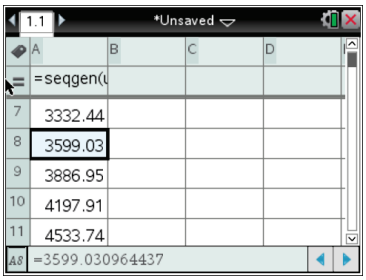
Question 17 **(6 marks)**

The recursive formula $T_{n+1} = 1.08T_n$, $T_0 = 2100$ can be used to calculate the value of an investment compounded annually for *n* years in the Farmers Bank of Western Australia.

(a) What is the annual interest rate? (1 mark)

Solution
8% per annum
Specific behaviours
✓ determines correct interest rate

(b) Calculate the value of the investment after seven years. (2 marks)

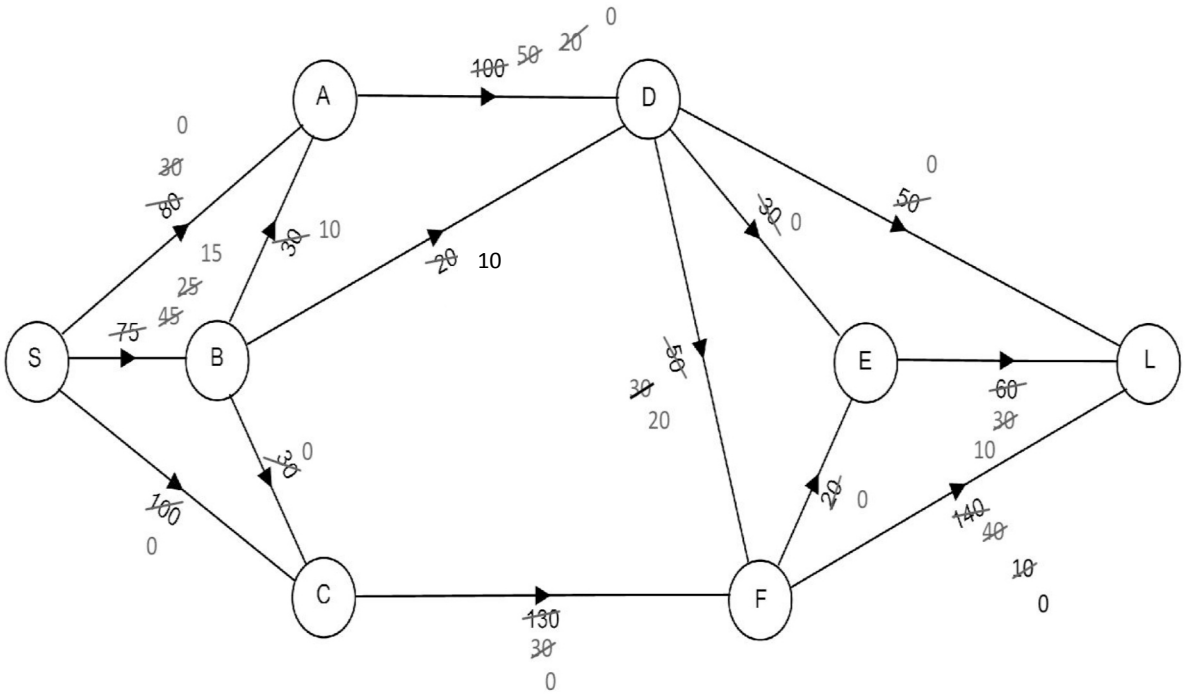
Solution	
$T_7 = 2100 \times (1.08)^7$ $T_7 = \$3599.03$ Accept \$3599	Or   
Specific behaviours	
✓ expresses T_7 in terms of T_0 ✓ calculates of T_7 correctly	✓✓ uses calculator to express T_7

(c) Determine the interest rate that would produce the same value for the investment above after a time of three years. (3 marks)

Solution
$r^3 \times 2100 = 3599.03$ $r = 1.1967$ Interest rate $= 19.67\%$ per annum
Specific behaviours
✓ sets up equations to calculate r ✓ solves equations for r ✓ expresses rate as a percentage or decimal (subtract 1 from r)

Question 18 **(5 marks)**

In a mining operation, ore is moved from a central stockpile 'S' to a loading station 'L' through an ore processing plant consisting of six processors ('A', 'B', 'C', 'D', 'E' and 'F') linked by a system of conveyor belts. The network below displays the operation with the arcs representing the conveyor belts. The number on each arc represents the maximum amount of ore, in tonnes per minute, that can be moved along that conveyor belt.

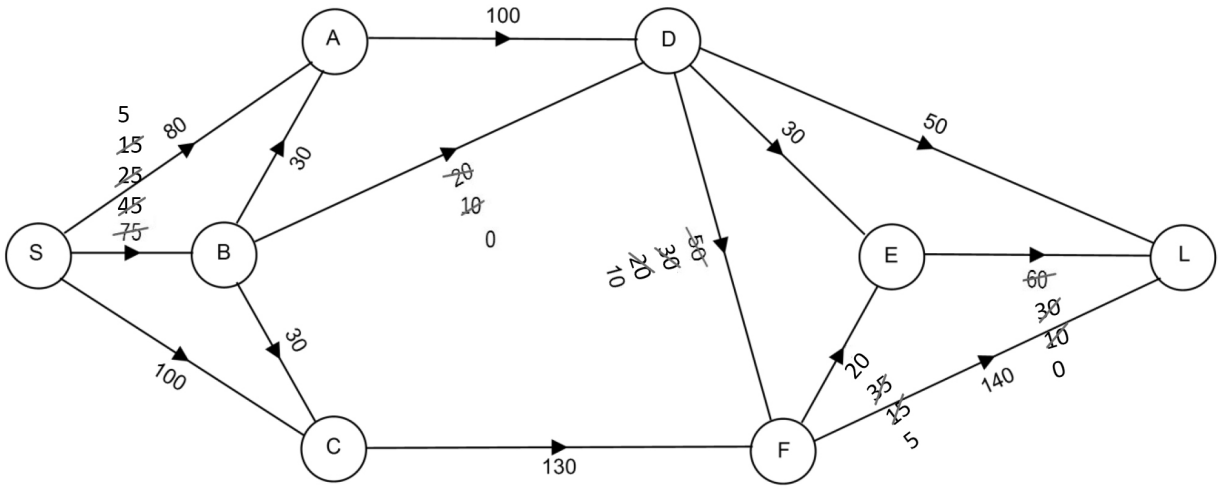


(a) What is the maximum amount of ore, in tonnes per minute, that can be moved from the stockpile S to the loading station L? Show systematic workings to allow your method to be checked. (3 marks)

Solution	
S, C, F, L	: 100
S, B, C, F, L	: 30
S, A, D, L	: 50
S, A, D, E, L	: 30
S, B, A, D, F, E, L	: 20
S, B, D, F, L	: 10
TOTAL	: 240 tonnes per minute
Specific behaviours	
✓ determines at least four paths with correct flow contribution ✓ determines all paths with correct flow contribution ✓ determines the correct maximum value or ✓ shows some appropriate cuts and capacity ✓ shows the minimal cut ✓ identifies the correct maximum amount of ore to the moved	

(b) What effect, if any, would there be on the maximum flow of ore from S to L if the capacity of the conveyor belt FE was increased by 15 tonnes per minute? Justify your answer.

(2 marks)



Solution
Additional flow of 10 tonnes per hour through the path. S, B, D, F, E, L path enables extra flow (or alternative path according to answer in part (a))
Specific behaviours
✓ identifies path or cut which allows for this increase (according to answer in part (a)) ✓ determines the correct increase (according to answer to part (a))

Question 19 **(11 marks)**

A theatre company performed for three weeks at a large venue capable of seating 4200 people. The attendances, in hundreds, at the evening performances, are shown in the following table.

Table 1: Moving averages

Day		Performance number (p)	Attendance (in hundreds)	Three-point moving average
First Week	Wednesday	1	20	
	Friday	2	16	20
	Saturday	3	24	22.7
Second Week	Wednesday	4	28	26
	Friday	5	26	29.3
	Saturday	6	34	30.7
Third Week	Wednesday	7	32	31.7
	Friday	8	A	34
	Saturday	9	41	

(a) Calculate the value of the missing entry marked by A. (2 marks)

Solution
$\frac{32+41+A}{3}=34 \quad \Rightarrow A=29$
Specific behaviours
✓ uses three-point moving average to determine an equation for A ✓ solves for A

(b) Given the seasonal index for the attendance at Friday performances is 84.64%, calculate the deseasonalised data (in hundreds) for Friday of Week 1, correct to **one** decimal place. (2 marks)

Solution
$16 \div 0.8464 = 18.9$
Specific behaviours
✓ selects correct day and week value for calculation ✓ divides by 0.8464 to calculate the data

Additional information about the theatre attendances is presented in the table below.

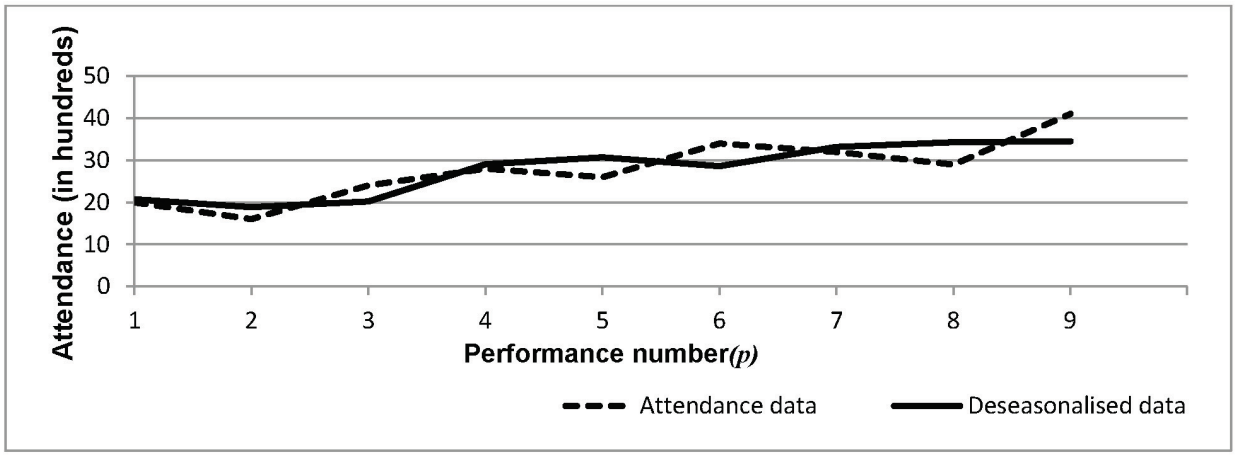
Table 2: Weekly and Percentage Means

		Performance number (p)	Attendance (in hundreds)	Weekly mean	Percentage of weekly mean
Week 1	Wednesday	1	20	20.0	100.0%
	Friday	2	16		80.0%
	Saturday	3	24		120.0%
Week 2	Wednesday	4	28	29.3	95.5%
	Friday	5	26		88.6%
	Saturday	6	34		115.9%
Week 3	Wednesday	7	32	34.0	94.1%
	Friday	8			85.3%
	Saturday	9	41		120.6%

(c) Show use of the average percentage method to calculate the seasonal index for the Saturday performances. (3 marks)

Solution
$\frac{120.0+115.9+120.6}{3}=118.83\%$ or 1.188
Specific behaviours
✓ uses average percentage method ✓ uses Saturday data residuals ✓ correctly averages percentages to calculate seasonal components

The attendance data and the deseasonalised data have been graphed on the axes below.



(d) (i) What has been the effect of deseasonalising the attendance data? (1 mark)

Solution
fewer fluctuations evident in the data or data is smoothed
Specific behaviours
✓ identifies fewer fluctuations or data smoothed

(ii) Describe the trend in attendance data over these three weeks. (1 mark)

Solution
attendance is increasing over time
Specific behaviours
✓ identifies attendance is increasing

(iii) Should the company extend its performances into a fifth week? Justify your choice of answer. (2 marks)

Solution
No – the theatre will have reached capacity. The trend is increasing but most people will have seen the show so attendance will fall.
Specific behaviours
✓ correctly answers 'no' ✓ states valid reason with reference to increasing trend/reaching capacity

Question 20 **(7 marks)**

CensusAtSchool Australia is a nationwide annual project that collects data about students. It provides a snapshot of the characteristics, attitudes and opinions of those students who have completed questionnaires. The tables below present information from the 2010 survey.

Table 1: Number and percentage of students by year level and sex who participated in the 2010 survey

Number of students				
Year level	**Female**	**Male**	**Total**	**% of total**
Yr 4 or below	203	213	416	1.9
Yr 5	1346	1262	2608	11.7
Yr 6	2097	2092	4189	18.8
Yr 7	1466	1464	2930	13.1
Yr 8	2059	1649	3708	**16.6**
Yr 9	2014	1741	3755	16.3
Yr 10	1461	1197	2658	11.9
Yr 11	793	706	1499	6.7
Yr 12	259	230	489	2.2
Other	32	35	67	**0.3**
Total	11 730	10 589	22 319	100

Table 2: Favourite takeaway foods by year level, 2010 Survey

Takeaway food	**Yr 4 or below**	**Yr 5**	**Yr 6**	**Yr 7**	**Yr 8**	**Yr 9**	**Yr 10**	**Yr 11**	**Yr 12**	**Other**
None	2.4	1.4	1.2	1.2	1.3	1.8	1.4	1.1	1.2	1.5
Chicken	6.7	6.8	7.3	6.3	7.3	8.4	9.4	10.1	9.4	14.9
Chips/fries	16.8	16.4	18.0	16.2	17.8	17.4	15.2	13.0	13.5	9.0
Fish	7.2	9.5	8.6	8.1	6.7	6.8	6.3	5.4	5.7	7.5
Fruit/fruit salad	4.3	2.6	2.4	2.4	2.1	2.0	1.8	1.9	2.0	4.5
Hamburgers	12.0	11.8	11.4	11.2	11.0	11.9	10.4	11.3	14.3	6.0
Kebabs/wraps	6.3	6.3	7.5	8.9	9.7	9.9	11.2	13.7	11.5	9.0
Noodle dishes	3.1	3.5	3.2	4.0	4.0	4.4	4.1	5.5	4.9	10.5
Pies/pasties	3.6	3.9	3.3	3.9	3.1	2.6	2.3	2.1	1.8	0.0
Pizza/pasta	22.1	21.6	22.4	22.7	21.9	21.2	23.1	21.4	21.5	11.9
Rice dishes	2.6	3.7	3.6	4.4	5.2	5.1	4.4	6.5	6.5	9.0
Rolls/sandwiches	1.9	1.4	1.5	1.9	1.9	2.2	3.6	2.6	2.5	0.0
Salads	1.2	2.8	2.0	1.7	1.6	1.4	1.8	0.9	1.8	4.5
Other	9.6	8.5	7.5	7.0	6.5	4.9	5.0	4.6	3.3	11.9

* The figures represent the percentages of the total number of students at each year level who participated in the 2010 survey.

By referring to Table 1 and Table 2, answer the following questions.

(a) Complete the Table 1 by adding the missing entries. (1 mark)

Solution
answers in bold as shown in table
Specific behaviours
✓ states percentages correctly – both answers must be correct for a mark to be awarded

(b) Which year level had the highest participation rate in the survey? (1 mark)

Solution
Year 6
Specific behaviours
✓ reads from two-way table correctly

(c) What percentage of the students who participated in the survey was female? (1 mark)

Solution
$\frac{11730}{22319} \times 100 = 52.6\%$
Specific behaviours
✓ identifies the number of females from the table 1 correctly ✓ calculates the percentage of females correctly

(d) Which was the least favoured takeaway food for Year 4 students? (1 mark)

Solution
salads
Specific behaviours
✓ identifies from Table 2 correctly

(e) How does this result for Year 4 students compare with that for Year 7 students? (1 mark)

Solution
same result – least favourite food for Year 7 students
Specific behaviours
✓ identifies that salad is least favourite

(f) Compare and comment on the popularity of pies/pasties of the students in the early years (up to Year 8) with those of the later years. Use evidence from the table to support your answer. (2 marks)

Solution
Pies/pasties were more popular in the early years. There were more than 3% of each year favouring them in the early years and in the later years this was always less than 3%
Specific behaviours
✓ identifies that pies/pastries are more popular in the early years ✓ compares the percentages differences

Question 21 **(8 marks)**

Pollution due to airborne lead has been reduced in Australia because of the requirement, since 1985, that new petrol-powered cars must use unleaded petrol.

The airborne lead data for Perth 1993-1999 are tabulated below.

Year (x)	1993	1994	1995	1996	1997	1998	1999
Average concentration of airborne lead (μg/m^3) (y)	0.86	0.60	0.39	0.24	0.15	0.13	0.08

(a) State the equation for the least-squares line that models these data. (1 mark)

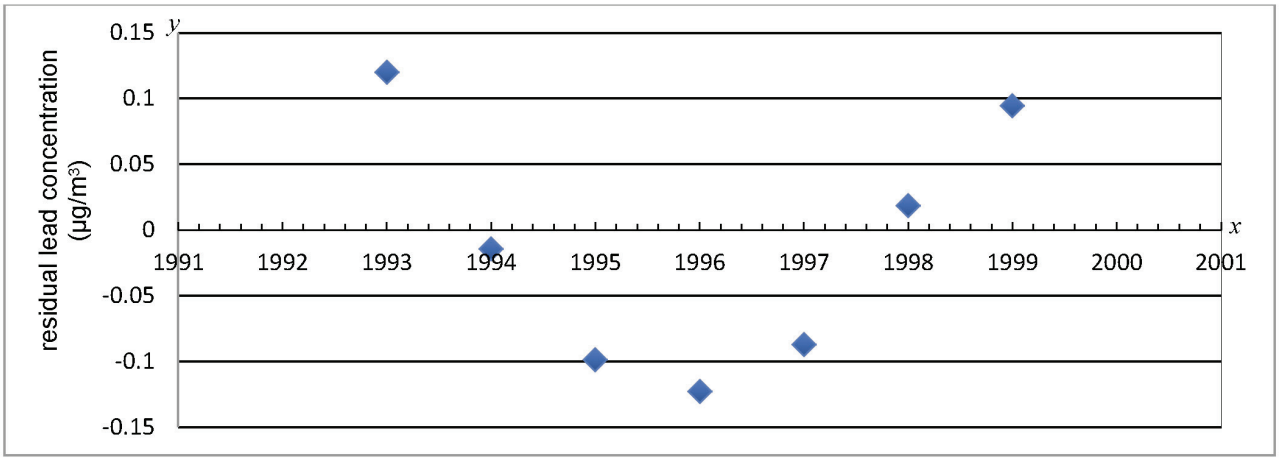
Solution
$\hat{y} = -0.1257x + 251.28,$ where $\hat{y}$ is the average lead concentration (g/m^3) and x is the year, 1993, 1994,... *or* $\hat{y} = -0.12571x + 0.85286,$ if x is '93 – 1, '94 – 2,... *or* $\hat{y} = -0.12571x + 1.1043,$ if x is '93 – 3, '94 – 4,...
Specific behaviours
✓ states a correct equation

(b) Determine the correlation coefficient for the data in the table above. (1 mark)

Solution
$r = -0.9430$ or -0.943 or -0.94
Specific behaviours
✓ determines the value of r

(c) Does the least-squares line determined in part (a) provide a good model for the 1993–1999 data? Justify your answer using evidence from the residual plot below.(2 marks)

Residual plot, annual average airborne lead concentration, Perth, 1993–1999



Solution
The line does not model the data well. The line does not follow the pattern of the data – as shown by the residuals which are not randomly distributed above and below zero as time increases.
Specific behaviours
✓ correctly states that the line does not model the data well ✓ gives one valid justification for conclusion

(d) A scientist suggested the following recursion model for the Perth lead-concentration data:

$$T_n = 0.6718\ T_{n-1},\ T_1 = 0.86\ ,$$

where $n = 1$ stands for 1993, $n = 2$ stands for 1994, and so on.

Assess whether this recursion model is appropriate for predicting the 2000 lead concentration in Perth air. Give **two** reasons to justify your assessment and state any assumptions that you made in arriving at your conclusion. (4 marks)

Solution
The recursion model is appropriate for predicting the 2000 lead concentration in Perth air. *Reasoning*: The recursion model is appropriate for prediction because it produces concentration values that are close the actual values (in particular for years immediately preceding 1998 and 1999) e.g. the 1998 actual value is 0.13 and the recursive value is 0.12 and the 1999 actual value is 0.08 and the recursive value is 0.08. *Assumption:* the decreasing trend continues (or assuming the percentage rate of decrease in lead concentration stays close to the same)
Specific behaviours
✓ states appropriateness of model ✓ justifies by giving reasons ✓ gives at least two numerical examples (or some mathematical reasoning) ✓ states assumptions using valid mathematical reasoning

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